ABSTRACT ALGEBRA

ITS HISTORY AND APPLICATIONS

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This thesis traces the development of abstract algebra beginning in 1801 when Gauss invented congruence, generally considered to be the earliest example of mathematical structure.

The first section deals with the men who were prime movers in the development of mathematical structure. Gauss invented congruence and proved the law of quadratic reciprocity. Soon afterwards a group of British mathematicians, principally Peacock, Gregory and De Morgan, proceeded to build a logical algebraic structure based on the natural numbers, and then to extend and modify the concept of such a structure to describe objects other than natural numbers and to allow operations other than conventional addition and multiplication. The German mathematician Hankel gave a thorough exposition of this work in 1876. From the idea of an algebraic structure with laws not necessarily the same as those of arithmetic came Hamilton's quaternions, Grassmann's more generalized algebras, and Boole's algebraic invariants. Boole is better known, however, for his invention of a mathematical structure for logic, and his so-called Boolean Algebras which give rise to the binary system and so make possible the digital computer. Cayley and Sylvester are credited with the extensive development of Boole's invariants,

and Cayley also with the representation of systems of equations by matrices and the development of the algebra of matrices.

The second section describes the development of the most important algebraic structures, groups, fields, rings, vector spaces and linear algebras. The group structure is defined, the four main types of groups are identified and examples of such structures given. The origin of group theory in the work of Lagrange is discussed, as is the work of Cauchy, Cayley, Galois, Sylow and Jordan. The structure of an algebraic field is defined, the principal example of which is the field of real numbers. The more general structure of the algebraic ring belonging to the integers is defined, and the concept of the ideal of a ring is explained. The vector space structure which grew out of the vectors of geometry and physics is defined, and lastly the linear algebras which combine features of both rings and vector spaces.

In the last section applications of algebraic structure in other areas are described. The first application was in logic, where it was used to provide a mathematical notation for expressing and solving problems in logic by means of algebraic manipulation. In other branches of mathematics, the use of invariants to classify geometries is described, and also the application of the field structure in the solution of a problem of antiquity, namely to prove

the impossibility of trisecting an angle using only straight-
edge and compass. In science, the association of an alge-
braic structure with the vectors of physics, the group of
symmetry operations important in crystallography, and the
application of group and matrix theory in quantum mechanics
are noted. Applications described in other disciplines are
those in music and in decision processes. The latter appli-
cation touches many other fields such as business and
psychology.

The final chapter comments on the importance of
abstract algebra today. Structures continue to become more
generalized, and thus more widely applicable, providing a
major unifying influence throughout the physical world.