A DEVELOPMENT OF THE NUMBER SYSTEM USING ORDERED NUMBER PAIRS

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This thesis emphasizes the fact that mathematics is a deductive science in which simple principles are assumed and where the superstructure follows as a logical consequence.

The thesis is a logical development of the number system, beginning with the natural numbers and expanding through the integers, reals, and then the complex and hypercomplex numbers. The properties of each set of numbers are examined, as are its limitations, and then the system is expanded.

The procedure followed to expand each number system is in accordance with Howard Fehr's statement:

> When new numbers are introduced or a new number system is created, we shall agree that the following steps must be taken.
>
> (1) The new numbers must be defined in terms of already learned numbers and understood relations.
>
> (2) Equality conditions for the new numbers must be defined, and if possible the numbers must be ordered.
>
> (3) The equality sign must be proved to be reflexive, symmetric, and transitive for the new numbers.
>
> (4) Operations of addition and multiplication must be defined for the new numbers.
>
> (5) The new numbers and the newly defined operations must be shown to obey the five fundamental laws.[1]

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[1]Howard Fehr, Secondary Mathematics, A Functional Approach for Teachers (Boston: D. C. Heath and Co., 1951), pp. 194-195.

Each set of numbers is studied to determine whether it is (1) a number system, and (2) a field.

To show that a set of numbers does form a number system, the five fundamental laws must be satisfied. These are: commutativity for addition and multiplication, associativity for addition and multiplication, and distributivity of multiplication over addition.

In order for a set of numbers to be a field, it must satisfy the requirements for a number system, and it must also be closed under the four operations of addition, subtraction, multiplication, and division.

The number systems studied in the thesis are measured against these properties, and a table of field properties is included for easy reference. The development from the natural numbers to the integers, and from the integers to the rationals, employs more rigor than does the development of the systems studied beyond these.

The main body of the thesis is a compilation of proofs for field properties. These proofs are largely analytic, although a discussion of both analytic and synthetic proof is included in the Appendix. Also several proofs from different sources are given for the distributive law for integers.

The ordered number pair technique is used in the thesis to limit the mathematical symbol while expanding the system. Thus the symbol (a,b) may represent an integer, a

rational, or a complex number according to the need. This simplifies the terms while analyzing the superstructure of the number system.